Table of Contents

1.0 INTRODUCTION .............................................................................................................................. .................................................. 4
2.0 SOURCES OF UNCERTAINTY .................................................................................................................. 4
3.0 INFLUENCE OF PARAMETERS ON SYSTEM RESPONSE ................................................................. 5
  3.1 Background .................................................................................................................................. 5
  3.2 Trajectory sensitivities .................................................................................................................. 5
  3.3 Quantifying parameter effects ...................................................................................................... 6
  3.4 Examples ...................................................................................................................................... 7
4.0 REDUCING UNCERTAINTY .............................................................................................................. 9
  4.1 Model structure ............................................................................................................................ 9
  4.2 Parameter estimation .................................................................................................................... 10
  4.3 Parameter conditioning ................................................................................................................. 10
5.0 IMPACT OF LOAD MODEL UNCERTAINTY .................................................................................. 11
  5.1 Overview .................................................................................................................................... 11
  5.2 Load-induced variations in qualitative behavior ........................................................................... 11
  5.3 Protection operation ..................................................................................................................... 12
6.0 APPROACHES TO REDUCING THE IMPACT OF UNCERTAINTY .................................................... 13
  6.1 Trajectory approximation using sensitivities ................................................................................. 13
  6.2 Probabilistic collocation method .................................................................................................. 15
  6.3 Grazing analysis ............................................................................................................................. 15
7.0 CONCLUSIONS ............................................................................................................................... 17
REFERENCES ......................................................................................................................................... 18
1.0 Introduction

Analysis of power system dynamic behavior requires models that capture the phenomena of interest, together with parameter values that ensure those models adequately replicate reality. It is important to distinguish between model fidelity and parameter accuracy. Models are always an approximation. In many cases, the level of approximation is determined by the nature of the study. For example, phasor-based models that are used for dynamic security assessment ignore electromagnetic transient phenomena. In other cases, however, model approximation is a matter of convenience, with the outcome not necessarily providing a good reflection of reality. Load modeling provides an example. It is common for the aggregate behavior of loads to be represented by a voltage dependent model, such as the ZIP model. This is a gross approximation, given the complex composition of loads on most distribution feeders. This deficiency is particularly evident in distribution systems that supply a significant motor load, as the ZIP model cannot capture the delayed voltage recovery associated with induction motors re-accelerating or stalling.

The choice of models is a decision that should be made based on knowledge of the actual system composition and the phenomena that are being studied. Determining parameters for those models, on the other hand, usually relies on comparison of model response with actual measured behavior. Parameter estimation processes seek to minimize the difference between measured and simulated behavior. An overview is provided in Section 4. Different choices for model structure will usually result in different parameter values. This is a consequence of the estimation process trying to compensate for unmodeled, or poorly modeled, effects. In all cases, the models and associated parameter sets are approximations, though the goal should always be to obtain the best possible approximations.

Load models are further complicated by the fact that load composition is continually changing. Even if it were possible to obtain a load model that was perfectly accurate at a particular time, it would be inaccurate a short while later. Developing load models is not a futile exercise though, as overall load composition tends to behave fairly predictably. For example, the composition of a residential feeder will (approximately) follow a 24 hour cycle. But, while composition from one day to the next may be roughly equivalent, morning load conditions may well differ greatly from those in the evening. Seasonal variations may be even more pronounced.

As mentioned previously, all models are approximate to some extent. Model structures for large dominant components, such as synchronous generators, are well established, as are procedures for determining the associated parameter values. Furthermore, parameter values for such devices remain fairly constant over their lifetime. Models that represent an aggregation of many distributed components are much more contentious though, given the inherent uncertainty in the overall composition of the model. This white paper focuses on uncertainty associated with load modeling. Similar issues arise in the modeling of other power system components though, with wind generation being a particularly topical example.

The white paper is organized as follows. Section 2 provides a discussion of the sources of uncertainty in load modeling. The influence of parameter variations on system response is discussed in Section 3. Reduction of uncertainty is considered in Section 4. Parameter estimation is reviewed, and parameter conditioning (identifiability) is discussed. The impact of load model uncertainty is considered in Section 5, while Section 6 reviews various techniques for assessing that impact.

2.0 Sources of Uncertainty

Loads form the major source of uncertainty in power system modeling. Loads are highly distributed, and quite variable, so detailed modeling is impossible. Aggregation provides the only practical approach to incorporating loads into power system studies. For static (power flow) analysis, the approximations inherent in aggregate load models are largely unimportant, as the composition of the load has little impact on results. On the other hand, load composition is very important in the analysis of system dynamic behavior. Different types of loads exhibit quite diverse responses to disturbances. For example, lighting loads vary statically (almost) with voltage, whereas motor loads exhibit dynamic behavior, perhaps even stalling. In fact, each different load type displays unique characteristics. Aggregate load models attempt to blend all those differing responses.
In many cases aggregate load models are required to represent loads that are widely distributed, physically and electrically. Because of this electrical separation, the voltages seen by loads may differ greatly. Such voltage differences may critically affect the response of loads to large disturbances, resulting in diverse load behavior. It is difficult for aggregate load models to capture such diversity. At best, those topological influences can only be crudely approximated.

Accounting for switching-type behavior in aggregate load models is also challenging. When residential air-conditioning compressor motors experience a voltage dip to around 0.6 pu, they almost instantaneously stall. This can be modeled as a mode switch, from running to stalled. As mentioned previously, voltage is usually not uniform across a distribution system. Therefore voltage dips may result in some compressor motors stalling and others not. As motors stall, the resulting high currents will further depress voltages, possibly inducing further stalling. The proportion of stalled motors will depend nonlinearly and temporally on many factors, including the severity of the initiating voltage dip, and the strength and topology of the distribution system. These attributes are difficult to capture, with any degree of certainty, in aggregate load models.

Other devices may also switch under disturbed voltage conditions. Contactors provide an example. They use an electromechanical solenoid to hold a switch in the closed position. When a disturbance depresses the voltage, the solenoid may not be able to hold the switch closed, resulting in unintended tripping of the associated load. The voltage threshold at which such action occurs varies widely. Precise modeling is not possible.

Looking to the future, a number of trends are likely to increase the level of uncertainty associated with aggregate load models. Distribution systems will see a greater penetration of distributed generation as fuel cells and solar cells, for example, become commercially viable. Plug-hybrid electric vehicles (PHEVs) will certainly gain in popularity, and may well become a significant feature of distribution systems. Not only do PHEVs present a load that moves from one location to another, but their vehicle-to-grid capability offers the possibility of highly dispersed generation. All these trends suggest that methods for assessing the impact of uncertainty are set to become increasingly important.

3.0 Influence of Parameters on System Response

3.1 Background

The time evolution of system quantities following a disturbance is referred to as a trajectory. For power systems, trajectories are driven by a system of switched differential-algebraic equations, with the switching required to capture events such as protection operation or limits being encountered. The details of this underlying model structure are not relevant to this report, and so are not included. A thorough discussion can be found in [1]. The concept of trajectories is important though, so a brief overview is provided.

The trajectory of a dynamical system depends on the initial conditions and the choice of parameter values. This dependence is expressed mathematically as the system flow, which can be written

\[ x(t) = \phi(t, x_0, \theta) \]

where the initial conditions are given by \( x_0 = x(0) \), and \( \theta \) denotes the parameters. For a particular choice of \( x_0 \) and \( \theta \), the point on the trajectory at time \( t \), denoted \( x(t) \) is given by evaluating the flow \( \phi \) at that time. Generally \( \phi \) cannot be written explicitly, but instead is generated numerically by simulation.

The report focuses on the impact of parameter uncertainty on the trajectory. It will be assumed that the initial conditions remain constant\(^1\). For notational convenience, the dependence of \( \phi \) on \( x_0 \) will therefore be ignored. Accordingly, trajectories will be given by

\[ x(t) = \phi(t, \theta). \]  

3.2 Trajectory sensitivities

Sensitivity concepts are generally associated with the linearization of an input-output relationship. Small changes in inputs map through the linearized relationship to small output changes. Trajectory sensitivities fit this framework by describing the changes in the trajectory (the output) resulting from perturbations in the underlying parameters and/or initial conditions (the inputs). They provide a linearization around the trajectory, as against small disturbance analysis

\(^1\)All subsequent analysis and techniques can be extended to incorporate variations in the initial conditions.
which builds on linearization around the equilibrium point. Trajectory sensitivity concepts are not new [2], though until recently progress on practical applications was impeded by:

- Computational inefficiency. Sensitivity to each parameter or initial condition required an additional full simulation.
- Non-smooth behavior. Sensitivities were not well defined for situations where events influenced behavior.

However both these limitations have recently been overcome, with efficient computation of trajectory sensitivities now possible for large-scale, non-smooth systems [3].

Trajectory sensitivities provide an insightful way of quantifying the effect that individual parameters have on overall system behavior [4]. A trajectory sensitivity is simply the partial derivative of the trajectory, or equivalently the flow, with respect to the parameters of interest,

\[ \Phi_i(t, \theta) = \frac{\partial \phi_i}{\partial \theta_j}(t, \theta) \]

where \( \phi_i \) refers to the \( i \)-th element of the vector function \( \phi \), or equivalently the \( i \)-th state, and \( \theta_j \) is the \( j \)-th parameter.

Trajectories are obtained by numerical integration, which generates a sequence of points at discrete time steps \( t_0, t_1, ..., t_N \) along the actual trajectory. The discretized trajectory will be described using the notation

\[ \mathbf{x} = [x(t_0), x(t_1), ..., x(t_N)] \]

(3)

Trajectory sensitivities can be calculated efficiently as a byproduct of numerical integration [3, 5, 6]. The corresponding discretized sensitivities can be written,

\[ \Phi_i(\theta) = \begin{bmatrix} \Phi_i(t_0, \theta) \\ \Phi_i(t_1, \theta) \\ \vdots \\ \Phi_i(t_N, \theta) \end{bmatrix} \]

(4)

Unfortunately, few commercial simulation packages currently provide trajectory sensitivity information. Approximate sensitivities must be generated by varying each parameter in turn by a very small amount, re-simulating, determining the difference in trajectories, and thus finding the sensitivity. The disadvantage of this method is that it is computationally expensive, and requires an additional simulation for each parameter.

### 3.3 Quantifying parameter effects

Trajectory sensitivities can be used directly to identify significant parameters in a model. Parameters that have a large associated trajectory sensitivity (for part or all of the simulation time) have a larger effect on the trajectory than parameters with smaller sensitivities. A 2-norm can be used to quantify this relative significance. Considering the sensitivity of the \( i \)-th system state (trajectory) to the \( j \)-th parameter, given by \( \Phi_{ij}(t, \theta) \), the 2-norm (squared) is given by

\[ \| \Phi_{ij}(t, \theta) \|_2^2 = \int_{t_0}^{t_N} \Phi_{ij}(t, \theta)^2 dt \]

(5)

where the period of interest is \( t_0 \leq t \leq t_N \). In terms of the discrete-time approximation provided by simulation, the equivalent 2-norm can be written

\[ \| \Phi_{ij}(\theta) \|_2^2 = \sum_{k=0}^{N} \Phi_{ij}(t_k, \theta)^2. \]

(6)
Parameters that have a relatively large (small) effect on the trajectory result in relatively large (small) values for these norms.

It should be kept in mind that the sensitivities \( \Phi(t, \theta) \) are computed for a single disturbance, and thus are applicable only for similar disturbances. Different forms of disturbances may excite the system in ways that accentuate the impact of other parameters. As a general rule, more severe disturbances yield higher sensitivities.

### 3.4 Examples

The examples throughout the report utilize the IEEE 39 bus system of Figure 1. All generators in this system were represented by a fourth order machine model [7], and were regulated by the IEEE standard AVR/PSS models AC4A and PSS1A [8]. All generator and network data were obtained from [9].

![IEEE 39 bus system](image)

**Figure 1: IEEE 39 bus system.**

#### 3.4.1 Parameter ranking

As mentioned previously, trajectory sensitivities provide a basis for ranking the relative influence of parameters. Large sensitivities imply that parameter variations have a large effect on behavior, whereas small sensitivities suggest behavior changes very little with parameter variation. In this example, trajectory sensitivities are used to rank the importance of voltage indices at all loads throughout the IEEE 39 bus system. A three-phase fault was applied at bus 16 at 0.1 sec, and cleared (without any line tripping) 0.2 sec later. A static voltage-dependent load model

\[
S_i(V) = P_i V_i^\eta_p + Q_i V_i^\eta_q
\]

was used for all loads, with \( \eta_p = \eta_q = 2 \) (constant admittance) in all cases.

The sensitivities of bus 16 voltage \( V_{16} \) to load indices \( \eta_p \) and \( \eta_q \) at all buses were computed in conjunction with the nominal trajectory. These trajectory sensitivities are provided in Figure 2, where the vertical axis gives the change in the pu voltage for a unity change in load index values. It is immediately clear that the real power index \( \eta_p \) for bus 20 has a much greater influence on behavior than all other indices. (The reason is that generator 5 is marginally stable for this disturbance scenario, and bus 20 lies on the corridor linking that generator to the rest of the system.) The loads at buses 4, 8 and 23 also display a reasonable, though certainly less pronounced, level of influence. Loads 4 and 8 are influential due to their large size. Load 23 has an important impact on the dynamics of generator 7. The influence of all other loads, for this disturbance scenario, is negligible. Of course a different disturbance could possibly highlight some other set of loads.
Field testing loads to determine their (approximate) voltage dependence is an expensive exercise. However, by utilizing trajectory sensitivities, the most important loads can be immediately identified, and attention focused accordingly. This use of trajectory sensitivities relates to parameter identifiability, and will be discussed further in Section 4.3.

### 3.4.2 Indicator of stressed conditions

It is shown in [4] that as systems become more heavily stressed, sensitivity to parameter variation increases significantly. This can be illustrated by continuing the previous example. The upper plot of Figure 3 shows the behavior of generator 5 angle (relative to generator 10) for a range of fault clearing times. (The fault clearing time used in the previous example was 0.2 sec.) The critical clearing time is 0.213 sec; slower clearing results in generator 5 losing synchronism. Notice that the angular deviations do not show a great increase, even though instability is imminent.

![Figure 2: Trajectory sensitivities for all load indices.](image)

The sensitivity of $V_{16}$ to the bus 20 load index $\eta_{p}$, for the same range of fault clearing times, is shown in the lower plot of Figure 3. The deviations exhibited by these trajectory sensitivities grow dramatically as critical conditions are approached. This behavior motivated the sensitivity related measures developed in [10,11] to predict conditions that induce marginal stability. Further work is required though to fully understand and exploit this phenomenon.
4.0 Reducing Uncertainty

4.1 Model structure

Load parameter uncertainty can be reduced by structuring models so that they adequately capture the physical characteristics of the actual loads. A ZIP model, for example, provides a poor representation of loads that include a significant proportion of air-conditioner motors. Attempting to replicate motor-induced delayed voltage recovery using such a model is futile. Tuning the ZIP parameters to best match one disturbance would provide no guarantee that the parameters were appropriate for another event. The WECC model of Figure 4, on the other hand, provides a versatile structure that is capable of representing various different load types. The issue with this latter model is one of identifying the multitude of parameters associated with the more complete model structure.
4.2 Parameter estimation

It is often possible to estimate parameter values from disturbance measurements. For example, simply measuring the active and reactive power consumed by a load during a disturbance may yield sufficient information to accurately estimate several model parameters. The aim of parameter estimation is to determine parameter values that achieve the closest match between the measured samples and the model trajectory.

Disturbance measurements are obtained from data acquisition systems that record sampled system quantities. Let a measurement of interest be given by the sequence of samples

\[ m = [m_0, m_1, \ldots, m_N] \]

with the corresponding simulated trajectory being given by

\[ x_i = [x_i(t_0), x_i(t_1), \ldots, x_i(t_N)] \]

which is the \( i \)-th row of \( x \) defined in (3). The mismatch between the measurement and its corresponding (discretized) model trajectory can be written in vector form as

\[ e(\theta) = x_i(\theta) - m \]

where a slight abuse of notation has been used to show the dependence of the trajectory on the parameters \( \theta \).

The best match between model and measurement is obtained by varying the parameters so as to minimize the error vector \( e(\theta) \) given by (10). It is common for the size of the error vector to be expressed in terms of the 2-norm cost,

\[ C(\theta) = ||e(\theta)||^2 = \sum_{k=0}^{N} e_k(\theta)^2. \]

The desired parameter estimate \( \hat{\theta} \) is then given by

\[ \hat{\theta} = \arg\min_{\theta} C(\theta). \]

This nonlinear least squares problem can be solved using a Gauss-Newton iterative procedure [12]. At each iteration \( j \) of this procedure, the parameter values are updated according to

\[ \Phi_i(\theta_j)^T \Phi_i(\theta_j) \Delta \theta^{j+1} = -\Phi_i(\theta_j)^T e(\theta_j)^T \]

\[ \theta^{j+1} = \theta^j + \alpha^{j+1} \Delta \theta^{j+1} \]

where \( \Phi_i \) is the trajectory sensitivity matrix defined in (4), and \( \alpha^{j+1} \) is a suitable scalar step size\(^2\).

An estimate of \( \theta \) which (locally) minimizes the cost function \( C(\theta) \) is obtained when \( \Delta \theta^{j+1} \) is close to zero. Note that this procedure will only locate local minima though, as it is based on a first-order approximation of \( e(\theta) \). However if the initial guess for \( \theta \) is good, which is generally possible using engineering judgement, then a local minimum is usually sufficient.

4.3 Parameter conditioning

The information content of a measured trajectory determines which parameters may be estimated. Parameters that have a significant effect on the trajectory are generally identifiable. Conversely, parameters that have little effect on trajectory shape are usually not identifiable.

When developing a parameter estimation algorithm, it is necessary to separate identifiable parameters from those that are not, in order to avoid spurious results. This can be achieved using a subset selection algorithm [14,15]. This algorithm considers the conditioning of the matrix \( \Phi_i^T \Phi_i \) that appears in (13). If it is well conditioned, then its inverse will be well defined, allowing (13) to be reliably solved for \( \Delta \theta^{j+1} \). On the other hand, ill-conditioning of \( \Phi_i^T \Phi_i \) introduces numerical difficulties in solving for \( \Delta \theta^{j+1} \), with the Gauss-Newton process becoming unreliable.

The subset selection algorithm considers the eigenvalues of \( \Phi_i^T \Phi_i \) (which are the square of the singular values of \( \Phi_i \)). Small eigenvalues are indicative of ill-conditioning. The subset selection algorithm separates parameters into those associated with large eigenvalues (identifiable parameters) and the rest which cannot be identified. The latter parameters are subsequently fixed at their original values.

\(^2\)Numerous line search strategies for determining \( \alpha \) are available in [13] and many other references.
Interestingly, the diagonal elements of $\Phi^T\Phi$ are exactly the values given by the 2-norm (6). If the trajectory sensitivities corresponding to parameters were orthogonal, then $\Phi^T\Phi$ would be diagonal, and separating the influences of parameters would be straightforward. This is not generally the case though, with the impacts of parameters often being partially correlated. For that reason, large values of (6) are not sufficient to guarantee parameter identifiability.

In summary, two situations lead to parameter ill-conditioning (non-identifiability). The first is where the trajectory sensitivities, corresponding to available disturbance measurements, are small relative to other sensitivities. This group of parameters cannot be estimated from available measurements. That may not be particularly troublesome though, if this is the only disturbance of interest, as their influence on behavior is negligible anyway. However, they may be influential for other disturbances. This should be assessed by considering a variety of viable disturbance scenarios. The second case arises when the trajectory sensitivities are highly correlated. Consequently, the influence of various parameters cannot be separated. This would be the situation, for example, when varying two parameters in unison gave no overall change in behavior. Both parameters are influential, but neither can be estimated without fixing the other. This dilemma may be resolvable by considering various disturbances, in the hope of finding cases where the parameters exert differing influences.

5.0 Impact of load model uncertainty

5.1 Overview

In terms of quantitative analysis, for example matching simulations with measurements, it is absolutely clear that accurate load modeling is vitally important. But for qualitative investigations, where the aim of dynamic simulation is to assess the likelihood of a certain disturbance scenario being stable or unstable, then the need for accurate load modeling is much reduced. In other words, if a system is stable (unstable) for a certain set of load model parameters, then it will most likely also be stable (unstable) for perturbed load models. Insights provided by trajectory sensitivities help explain this conjecture. In fact, for such qualitative assessment, it is more important to know the sensitivity of behavior to load parameters, than to precisely know the parameter values.

A caveat is required though. Most power system failures are not initiated by instability [16], though instability is frequently a consequence. Rather, it is more common for an initiating (relatively minor) disturbance to escalate through reactionary protection operation. Examples of such reactionary effects include zone 3 distance protection unnecessarily tripping feeders, and volts/hz relays tripping generators. The subsequent weakening of the system may induce further protection operation, leading to cascading system failure. It has been found from disturbance analysis that load modeling can be very important in predicting such reactionary protection behavior [17].

Protection is binary; either the system encounters the operating characteristic initiating a trip, or it does not encounter the characteristic and the component remains in service. The bounding case, separating protection operation from non-operation, corresponds to the trajectory *gazing* (just touching) the operation characteristic [18]. Parameter sets that induce grazing are pivotal, in that they divide parameter space into regions that exhibit vastly different behavior [19]. It follows that in potential grazing situations, where reactionary protection operation may or may not occur, special care should be given to understanding the influence of load parameter variations.

5.2 Load-induced variations in qualitative behavior

Previous analysis and examples have suggested that load models have negligible qualitative influence on the behavior of systems that are robustly stable [4]. This will be further illustrated using the IEEE 39 bus system of Figure 1, though in this case the disturbance scenario involves a solid three-phase fault on line 16-21, at the bus 21 end. The fault was cleared after 0.15 sec by tripping the faulted line. That left buses 21 and 23, and generators 6 and 7, radially fed over line 23-24.

The load composition at buses 23 and 24 was modeled parameterically by

$$S_{\text{tot}} = vS_v + (1-v)S_{\text{ind}}$$ (15)

where $S_{\text{tot}}$ is the total complex power of the load, $S_v$ is the static voltage dependent part of the load given by (7), and $S_{\text{ind}}$ is the complex power demanded by the induction motor component. The dynamics underlying $S_{\text{ind}}$ are typically described by a third order differential equation model [20]. For this example, both $v_{23}$ and $v_{24}$ were nominally set to...
0.5. In other words, both loads were composed of 50% static voltage dependent load and 50% induction motor load. The static load component was modeled as constant admittance, while the induction motor component used parameter values from [20, p. 305], with appropriate per unit scaling.

Figure 5: Influence of load parameter perturbations with increased system stress.

The response of generator 6 angle (relative to generator 1), under the nominal load conditions, is shown as a dashed line in Figure 5. The load composition parameters \( \nu_{23} \) and \( \nu_{24} \) were then varied between extremes of 0 and 1. The corresponding behavior is shown as thick solid lines in Figure 5. Notice that this large variation in load composition has negligible effect on the qualitative form of the response.

The fault clearing time was then increased to 0.18 sec, quite close to the critical clearing time of 0.18375 sec. Nominal behavior is again shown as a dashed line, with behavior corresponding to extremes in \( \nu_{23} \) and \( \nu_{24} \) shown as thinner solid lines. In this case, it turns out that reduction of \( \nu_{24} \) to 0 has a marked effect on the qualitative form of the response; the system is only just stable.

This example supports the hypotheses that load modeling only becomes important qualitatively when the system is close to instability, and that proximity to instability can be detected by high sensitivity.

5.3 Protection operation

The previous example showed that for unstressed systems, load composition has negligible effect on the qualitative form of behavior. However that example did not take account of protection. In this example, zone 3 protection at the bus 23 end of line 23-24 is considered. Figure 6 shows the separation\(^3\) between the zone 3 mho characteristic [21] and the apparent impedance seen from bus 23. The dashed line was obtained for a fault clearing time of 0.15 sec, and used the nominal set of load parameters. It remains above zero, suggesting the zone 3 characteristic is not entered.

---

\(^3\)This distance goes negative when the apparent impedance enters the mho characteristic.
Uncertainty was introduced into the load composition parameters $\nu_{23}$ and $\nu_{24}$. They were assumed normally distributed, with mean of 0.5 and standard deviation 0.1. A Monte-Carlo process was used to generate thirty random parameter sets, with the resulting trajectories shown in Figure 6. The figure also shows the 95% confidence interval. Notice that it is quite probable for trajectories to pass below zero, suggesting the possibility of a zone 3 trip. Knowledge of the load composition is therefore very important in this case.

6.0 Approaches to reducing the impact of uncertainty

6.1 Trajectory approximation using sensitivities

By expanding the flow (1) as a Taylor series, and neglecting the higher order terms, trajectories arising from perturbing parameters by $\Delta \theta$ can be approximated as

$$\phi(t, \theta + \Delta \theta) \approx \phi(t, \theta) + \Phi(t, \theta) \Delta \theta$$

where $\phi(t, \theta)$ is the trajectory obtained using the nominal set of parameters $\theta$, and the corresponding trajectory sensitivities are given by $\Phi(t, \theta)$. If the perturbations $\Delta \theta$ are relatively small, then the approximation (16) is quite accurate. This accuracy is difficult to quantify though. It is shown in [4] that the higher order terms neglected in (16) become increasingly significant as the system becomes less stable. Nevertheless, the approximations generated by (16) are generally quite accurate.

The affine nature of (16) can be exploited to establish two straightforward approaches to mapping parameter uncertainty through to bounds around the nominal trajectory [4]. The first approach assumes that each uncertain parameter is uniformly distributed over a specified range. Multiple uncertain parameters are therefore uniformly distributed over a multidimensional hyperbox. As time progresses, the affine transformation (16) distorts that hyperbox into a multidimensional parallelepiped. A simple algorithm is proposed in [4] for determining the vertices of the time-varying parallelepiped that correspond to worst-case behavior.

The example of Section 5.3 can be used to illustrate this process. An uncertainty of $\pm 0.2$ was assumed in both load composition parameters, so that

$$0.3 \leq \nu_{23}, \nu_{24} \leq 0.7.$$  

It can be expected that 95% of trajectories lie within that bound.
Worst-case analysis was used to explore bounds on behaviour, and in particular to determine whether this uncertainty could affect conclusions regarding protection operation.

The example again focuses on zone 3 protection at the bus 23 end of line 23-24. The dashed line in Figure 7 was obtained using the nominal set of load parameters. As mentioned before, it suggests the zone 3 characteristic is not entered. Based on this nominal trajectory, sensitivities indicated that over the time frame of interest, where the trip signal approached zero, worst behaviour (lowest dip) occurred for load indices $\nu_{23} = 0.7$ and $\nu_{24} = 0.3$. Best behaviour (least dip) occurred for $\nu_{23} = 0.3$ and $\nu_{24} = 0.7$. The corresponding approximate (sensitivity derived) bounds on behaviour are shown as solid lines in Figure 7. The true (simulated) bounds are shown as dash-dot lines. The sensitivity-based predictions are very accurate over this crucial time period. Every selection of $\nu_{23}$ and $\nu_{24}$ from the range (17) results in a trajectory that lies within the bounds shown in Figure 7. Notice that the lower bound passes below zero, indicating the possibility of a zone 3 trip.

![Figure 7: Zone 3 protection on line 23-24, worst-case bounds for $0.3 \leq \nu_{23}, \nu_{24} \leq 0.7$.](image)

Often parameter values are not uniformly distributed over the range of uncertainty, but are better described by a normal distribution. Under those conditions, worst-case analysis gives a conservative view of parametric influences. Less conservatism is achieved with probabilistic assessment.

A probabilistic approach to assessing the influence of uncertainty assumes $\theta$ is a random vector with mean $\mu$ and covariance matrix $\Sigma$. It follows that deviations

$$\Delta \theta = \theta - \mu$$  \hspace{1cm} (18)

have zero mean and covariance $\Sigma$. The nominal flow and corresponding trajectory sensitivities are generated with parameters set to $\mu$. Perturbations in the trajectory at time $t$ are given (approximately) by

$$\Delta x(t) = \Phi(t, \theta) \Delta \theta.$$  \hspace{1cm} (19)

It follows from basic statistical properties [22] that perturbations in state $i$ will have mean

$$E[\Delta x_i(t)] = \Phi_i(t, \theta) E[\Delta \theta] = 0$$  \hspace{1cm} (20)

and variance

$$\text{Var}[\Delta x_i(t)] = \Phi_i(t, \theta) \Sigma \Phi_i(t, \theta)^T.$$  \hspace{1cm} (21)

Furthermore, if the elements of random vector $\Delta \theta$ are statistically independent, then $\Sigma$ will be diagonal with elements $\sigma_1^2, \ldots, \sigma_n^2$. In this case, (21) reduces to

$$\text{Var}[\Delta x_i(t)] = \sum_{j=1}^{n} \Phi_{ij}(t, \theta)^2 \sigma_j^2.$$  \hspace{1cm} (22)
Referring back to the example of Section 5.3, the two load composition parameters \( v_{23} \) and \( v_{24} \) each had mean \( \mu = 0.5 \) and variance \( \sigma^2 = 0.01 \). Equation (22) was used to determine the variance of the zone 3 protection signal at each time step along the trajectory. The bounds shown by solid lines in Figure 6 are constructed from points that are plus/minus 1.96 times the standard deviation away from the nominal trajectory. The choice of 1.96 corresponds to the 95% confidence interval.

### 6.2 Probabilistic collocation method

The probabilistic collocation method (PCM) provides a computationally efficient approach to building an approximate relationship between random variables and outputs that depend upon those variables. In assessing the impact of parameter uncertainty, it is assumed that the parameters of interest satisfy given probability density functions \( f(\lambda) \). The desired outputs are obtained by running a simulation for each randomly chosen set of parameters. Any feature of the trajectory could be chosen as an output, for example the values of states at certain times, and/or the maximum voltage dip.

This section provides an overview of PCM. More complete details are presented in [23]. In order to simplify notation, the discussion will assume a single uncertain parameter. The ideas extend to larger numbers of parameters, though with increased computations.

For a given probability density function \( f(\lambda) \), a set of orthonormal polynomials \( h_i(\lambda) \) can be determined. The subscript \( i \) refers to the order of the polynomial, and orthogonality is defined in terms of the inner product

\[
\langle h_i(\lambda), h_j(\lambda) \rangle = \int f(\lambda) h_i(\lambda) h_j(\lambda) d\lambda.
\]

Underlying PCM is the assumption that the uncertain parameter \( \lambda \) and the output of interest are related by a polynomial \( g(\lambda) \) of order \( 2n - 1 \). This is generally not strictly true, though such polynomial approximation is not unusual. Given this "true" relationship \( g(\lambda) \) between parameter and output, PCM determines a lower order polynomial \( \hat{g}(\lambda) \) such that the mean value for \( \hat{g}(\lambda) \) coincides with that of \( g(\lambda) \),

\[
E[\hat{g}(\lambda)] = E[g(\lambda)].
\]

If \( g(\lambda) \) is of order \( 2n - 1 \), then \( \hat{g}(\lambda) \) has order \( n - 1 \), and can be written in terms of the orthonormal polynomials \( h_i(\lambda) \) as

\[
\hat{g}(\lambda) = g_0 h_0(\lambda) + g_1 h_1(\lambda) + \ldots + g_{n-1} h_{n-1}(\lambda).
\]

(23)

The coefficients \( g_0, \ldots, g_{n-1} \) are obtained by solving

\[
\begin{bmatrix}
  g(\lambda_1) \\
  \vdots \\
  g(\lambda_n)
\end{bmatrix} =
\begin{bmatrix}
  h_{n-1}(\lambda_1) & \cdots & h_0(\lambda_1) \\
  \vdots & \ddots & \vdots \\
  h_{n-1}(\lambda_n) & \cdots & h_0(\lambda_n)
\end{bmatrix}
\begin{bmatrix}
  g_{n-1} \\
  \vdots \\
  g_0
\end{bmatrix}
\]

(24)

where the \( \lambda_i \) are the roots of \( h_n(\lambda) \).

In summary, for a given probability density function \( f(\lambda) \) for the uncertain parameter, PCM requires the following computations. The set of orthonormal polynomials \( h_0, \ldots, h_n \) corresponding to the given \( f(\lambda) \), can be obtained using a straightforward recursive algorithm [24]. The roots of \( h_n(\lambda) \) provide the values \( \lambda_1, \ldots, \lambda_n \) which are used in simulations to obtain the output values \( g(\lambda_1), \ldots, g(\lambda_n) \). Also, \( h_0, \ldots, h_{n-1} \) are evaluated at \( \lambda_1, \ldots, \lambda_n \) to establish the matrix in (24), which is subsequently inverted to obtain the coefficients \( g_0, \ldots, g_{n-1} \). These coefficients are used in (23) to give the desired lower-order approximation \( \hat{g}(\lambda) \).

### 6.3 Grazing analysis

As discussed in Section 5, most power system disturbances escalate through events such as operation of protection devices. In order to assess vulnerability to events, triggering conditions such as protection operating characteristics can be conceptualized as hypersurfaces in state space. A trajectory that passes close by a hypersurface, but does not encounter it, will not initiate an event. On the other hand, if the trajectory does encounter the hypersurface, an event will occur, possibly with detrimental consequences.
Trajectories \( \phi(t, \theta) \) are parameter dependent. For a certain set of parameters, the trajectory may miss the event triggering hypersurface. The hypersurface may be encountered for a different set of parameters though. These two situations are separated by trajectories that only just touch the hypersurface. This is illustrated in Figure 8. The critical condition, which separates two different forms of behavior, is referred to as \textit{grazing} [18].

![Illustration of grazing](image)

Figure 8: Illustration of grazing.

It is shown in [18] that grazing conditions establish a set of algebraic equations that can be solved using a Newton process. Each iteration of the Newton algorithm requires simulation to obtain the trajectory and associated sensitivities. Such solution processes are known as shooting methods [25]. Full details for grazing applications can be found in [18,19].

Referring to the example illustrated in Figures 6 and 7, grazing analysis can be used to determine the smallest changes in parameters \( \nu_{23} \) and \( \nu_{24} \) that cause the apparent impedance trajectory to just touch the mho characteristic. Such information provides another mechanism for assessing whether system behavior is robust to uncertainty in these parameters.

![Grazing resulting from two different sets of parameters](image)

Figure 9: Grazing resulting from two different sets of parameters.

This situation is illustrated in Figure 9. Setting \( \nu_{23} = 0.67 \) while holding \( \nu_{24} = 0.5 \) results in grazing. Alternatively, grazing also occurs when \( \nu_{23} = 0.5 \) and \( \nu_{24} = 0.34 \). Other combinations of perturbations in \( \nu_{23} \) and \( \nu_{24} \) also result in grazing. Figure 10 provides a parameter-space view of grazing conditions. The two previous cases have been augmented with a third grazing scenario, where \( \nu_{23} = 0.596 \) and \( \nu_{24} = 0.404 \). Further grazing points could be
found, using a continuation process, to establish a line in parameter space. Proximity to that line would suggest vulnerability to grazing, and hence to event triggering. This is illustrated in Figure 10. The point corresponding to the nominal parameter values $\nu_{23} = \nu_{24} = 0.5$ is shown, along with a dashed line that indicates uncertainty of $\pm 0.15$ in $\nu_{23}$ and $\nu_{24}$. The region describing parameter uncertainty overlaps the line of grazing points. This suggests a finite probability that the mho characteristic will be encountered, and hence that protection will operate.

This grazing-based approach to assessing robustness to uncertainty can be extended to an arbitrary number of parameters. The information derived from such analysis is useful for exploring the relative impact of uncertainty in the different parameters. For example, it may show that a small variation in one of the parameters may induce grazing, whereas a much larger variation could be tolerated in a different parameter. These concepts are explored in [19] in the context of power electronic circuits. Adaptation to power system applications is conceptually straightforward, though has not yet been undertaken.

![Figure 10: Parameter-space view of vulnerability to grazing conditions.](image)

### 7.0 Conclusions

All models are an approximation, to some extent. Uncertainty in model-based analysis is therefore unavoidable. Model design should take into account the nature of the phenomena under investigation, with well designed models minimizing the impact of unmodeled effects and of uncertainty. In power systems, the major source of uncertainty arises from the modeling of loads. Accurate modeling is particularly challenging due to the continual variation in load composition.

Trajectory sensitivities provide a numerically tractable approach to assessing the impact of uncertainty in parameters. Such sensitivities describe the variation in the trajectory resulting from perturbations in parameters. Small sensitivities indicate that uncertainty in the respective parameters has negligible impact on behavior. Large sensitivities, on the other hand, suggest that the respective parameters exert a measurable influence on behavior. It is important to minimize the uncertainty in the latter group of parameters. This can be achieved by estimating parameter values from measurements of system disturbances. The parameter estimation process seeks to minimize the difference between measured behavior and simulated response. This difference can be formulated as a nonlinear least squares problem, with the solution obtained via a Gauss-Newton process. Trajectory sensitivities provide the gradient information that underlies that process.

The impact of uncertain parameters is generally not significant for systems that are unstressed. As the stability margin reduces, however, system behavior becomes much more sensitive to parameter perturbations. It is particularly important to consider cases that are on the verge of protection operation. In such cases, uncertainty may make the difference between protection operating or remaining inactive, with the consequences being vastly different.
Various numerical techniques are available for assessing the impact of parameter uncertainty. Trajectory sensitivities can be used to generate approximate trajectories, which in turn allow parameter uncertainty to be mapped to a bound around the nominal trajectory. The probabilistic collocation method can be used to determine (approximately) the statistical distribution associated with important features of a trajectory. This method can also be used to establish an uncertainty bound around the nominal trajectory. The likelihood that uncertain parameters may induce undesirable events, such as reactionary protection operation, can be assessed using techniques that build on grazing concepts.

References


