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Testing Branching Process Estimators of Cascading Failure with Data from a Simulation of Transmission Line Outages

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We suggest a statistical estimator to quantify the propagation of cascading transmission line failures in large blackouts of electric power systems. We use a Galton-Watson branching process model of cascading failure and the standard Harris estimator of the mean propagation modified to work when the process saturates at a maximum number of components. If the mean number of initial failures and the mean propagation are estimated, then the branching process model predicts the distribution of the total number of failures. We initially test this prediction on failure data generated by a simulation of cascading transmission line outages on two standard test systems. We discuss the effectiveness of the estimator in terms of how many cascades need to be simulated to predict the distribution of the total number of line outages accurately.

KEY WORDS: Blackout risk; branching process; cascading failure; electric power transmission system; infrastructure

1. INTRODUCTION

Cascading failure is a sequence of dependent failures that successively weaken a system. In electric power transmission systems, cascading failure is the main way that blackouts become more widespread. For example, the August 2003 blackout affecting 50 million people spread to a sizable region of Northeastern America by cascading.(1) Other examples are the July and August 1996 blackouts of the Western power system of North America(2) and the November 2006 European blackout in which failures propagated from Germany to Southern Europe.(3) In these examples, a small initial disturbance spread to a large blackout by cascading. Some blackouts with a large initial disturbance such as caused by extremely bad weather may also spread further via cascading. Cascading failure is of great interest in the risk analysis of several of the interconnected infrastructures that underpin our society, but here we focus on initial testing of methods of analyzing cascading failure in electric power transmission systems. Indeed, the impact of large blackouts on society is a good motivation for the analysis of cascading failure. An initial review of methods for cascading failure in electric power systems is in Reference 4.

Electric power transmission systems are meshed networks at high voltages that form the backbone of the electric power system.(5) They can be of continental scale with thousands or tens of thousands of transmission lines and nodes. The components of transmission systems include transmission lines, transformers, substations, and protection, control, communication, and computing equipment. In our terminology, the "failure" of a component can include automatic or manual de-energizing of the component so that it is not damaged but is unavailable to transmit power, or a component malfunctioning or

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becoming damaged. Other types of failures are human errors or errors in software or operational procedures.

There are many different mechanisms of cascading in electric power transmission systems by which one failure can cause other failures. Large blackouts are typically complicated sequences of cascading failures that combine several of these mechanisms .[1–3,6] For a readable account of the formidable complexities of blackouts, see Reference 1, which describes cascading line failures as well as many other cascading processes. However, one common feature of blackouts is the successive failure of transmission lines. Moreover, the number of transmission lines failed is one measure of blackout size. (The number of transmission lines failed is not a measure of direct impact to society as is energy unserved or customers disconnected, but it does provide an accessible measure of blackout size internal to the power system that is useful to utilities.) Thus, transmission line failures are useful diagnostics in monitoring the progress and extent of blackouts.

The traditional way to address cascading failure in electric power transmission systems is to design and operate the system so that a chosen subset of severe initial failures do not cause subsequent failures. This approach tends to prevent some cascading failures, but other cascading failures can still occur. We suggest that one complementary approach would be to monitor and limit the subsequent propagation of a cascade of failures after the initial disturbance. To do this, it would be very useful to be able to quantify how much failures propagate. In this article, we propose an estimator of the mean failure propagation λ that is based on a branching process model of cascading failure. The estimator should work when the number of components that can fail "saturates" at a given maximum number of components. The reasons for modeling saturation are discussed in Section 2.2.

To develop methods toward quantifying the risk of cascading failure, we need to predict the probability distribution of the number of transmission line failures as one measure of the distribution of blackout size. We start with cascading failure data produced by the OPA[2] simulation of power system transmission line overloads described in Section 4 and Reference 7. These cascading failure data describe how many simulated lines trip in successive stages of each cascade. We show how to estimate the mean propagation λ and the mean initial line failures θ from these cascading failure data. These estimated quantities are the parameters of a Galton-Watson branching process model of cascading failure, and substituting the estimates for these parameters in an analytic formula predicts the probability distribution of the number of transmission line failures. We initially test this combined use of the estimates and the branching process model to predict the probability distribution of the number of transmission line failures. The testing compares this predicted probability distribution with the empirical probability distribution of the number of transmission line failures produced by exhaustively running the OPA simulation.

Branching process models are an obvious possible choice of stochastic model to capture the gross features of cascading blackouts because they have been developed and applied to other cascading processes such as genealogy, initial spread of epidemics and cosmic rays,[8] and avalanches in idealized sandpiles.[9] Thus estimation of the mean propagation λ is already established in cascading failure in other applications.[10] The first suggestion to apply branching processes to cascading failure in blackouts is in Reference 11 and subsequent applications appear in References 12–17. One difference between stochastic modeling with branching processes of cascading in blackouts and cascading in the previous applications is the many and complicated mechanisms by which failures propagate in blackouts. In this article, we test the branching process modeling of cascading with data produced by a simulation of one of these mechanisms, namely, cascading transmission line overloads. Similar testing with data produced by simulations of other blackout mechanisms or combinations of mechanisms is future work.

We now discuss the evidence that branching processes can be useful approximations to some of the gross features of cascading blackouts. The idealized probabilistic model of cascading failure[6] describes a general cascading process in which component failures weaken and further load the system so that subsequent failures are more likely. This cascading failure model and variants of it can be well approximated by a Galton-Watson branching process with each failure giving rise to a Poisson distribution of failures in the next stage.[11,18] Moreover, some features of this cascading failure model are consistent with results from cascading

[2] OPA stands for Oak Ridge National Laboratory, Power Systems Engineering Research Center at the University of Wisconsin, and the University of Alaska and indicates the institutions collaborating to devise the simulation.

failure simulations.[7,12,19–21] All of these models can show criticality and power-law regions in the distribution of failure sizes or blackout sizes consistent with North American data[22,23] and data from other countries.[24] The distribution of the number of high-voltage transmission lines lost in North American contingencies from 1965 to 1985[25] also has a heavy tail distribution that is fairly close to a power law.[26] The first work fitting or testing branching process models with observed blackout data is in References 13 and 16.

Initial work toward this article appeared in parts of the conference article[14] and Wierzbicki's Master's thesis.[27] The new contributions of this article relative to this initial work are to introduce a new estimator for the propagation of failures, prove that the new estimator is unbiased and examine its variance, analyze the number of cascades needed for statistical accuracy, and substantially rework and rewrite the expression of the ideas. Some of these contributions are also described in parts of Kim's Master's thesis[28] and summarized in the conference paper.[29] This article also includes new processing of the results on the IEEE 118 bus test system and new results on the IEEE 300 bus test system.

In a Galton-Watson branching process,[8,30] the failures are produced in stages. The process starts with Z_0 failures at stage zero to represent the initial disturbance. The failures in each stage independently produce further failures in the next stage according to a probability distribution called the offspring distribution. The offspring distribution has mean λ. That is, each failure in each stage propagates to produce an average of λ failures in the next stage.

The eventual behavior of the branching process is governed by the mean propagation λ. In the subcritical case of $\lambda < 1$, the failures will die out (i.e., reach and remain at zero failures at some stage) and the mean number of failures in each stage decreases exponentially. In the supercritical case of $\lambda > 1$, although it is possible for the process to die out, often the failures increase exponentially until the system size or other saturation effects are encountered.

At the critical case of $\lambda = 1$, the branching process has a power-law region of the probability distribution of number of failures with exponent -1.5 (Otter's theorem[8]). A corresponding power-law region can be observed in the distributions of number of failures in the cascading failure model[6] and in the distribution of blackout size in blackout models[7,20,21] when the system has a particular loading called the critical loading. The implications of the power-law region are that the risk of large blackouts is comparable to or even exceeding the risk of small blackouts (Section 2 and Reference (31)) (here blackout risk is the product of blackout probability and blackout cost). This observation justifies the study of large blackouts; an exponential tail in the distribution of blackout size would imply that large blackouts have negligible risk and that a risk-based analysis would ignore large blackouts. Moreover, at criticality the mean blackout size starts to increase more rapidly and above criticality there is an increasing risk of large blackouts. The terminology of criticality comes from statistical physics and does not, at this stage of knowledge about blackout risk, necessarily imply improper power system operation. Indeed, there is some evidence that power systems may organize themselves to near critical loading in response to strong societal forces balancing economic use of the transmission system and reliability.[22,32]

One requirement for the failure data in order to estimate λ is that the failures be grouped into stages. The estimator we propose depends on the number of failures in the stages and particularly on the total number of failures and the number of failures in the initial and final stages. Many cascading failure simulations naturally produce failures in stages as the simulation iterates. However, if the method is applied to real data, the problem of grouping the data into stages must be addressed. References 13 and 16 use simple methods of grouping failures according to their timing.

One direct way to estimate the probability distribution of number of line failures is simply to run the simulation or record real blackout data until sufficient data are accumulated to estimate the empirical probability distribution. This is straightforward but requires a large number of simulations or an impractically long observation time. If the distribution of line failures is near criticality and therefore has a power-law character, the empirical probability distribution requires many observations to determine its form for the larger blackouts. We discuss the efficiency of predicting the distribution of the number of line failures via the branching process model compared to estimating the distribution empirically in Section 6.

More generally, if high-level probabilistic models, such as branching processes, can be established for cascading failure of electric power transmission systems, this would allow efficient estimation of the model parameters and hence better estimation of blackout risk. These possibilities are further indicated in Reference 33. One general approach would start with the probability distribution of the size of initial failures, which can be evaluated for several measures of size using conventional risk analysis, and then would determine the extent to which these initial failures propagate in a cascading process to a more widespread blackout. This amounts to estimating the probability distribution of the cascading blackout size for various measures of blackout size. The probability distribution of the cascading blackout size could then be combined with estimates of blackout cost to estimate the probability distribution of blackout risk. Moreover, some of the model parameters, such as the mean propagation λ considered in this article, could give insight into the cascading process and be useful in monitoring real or simulated power systems. For example, mean propagation λ could be a measure of system resilience to cascading and changes in λ over time could be monitored similarly to the other blackout statistics analyzed in Reference 34.

In summary, the goals of the article are to use a Galton-Watson branching process model of cascading failure and failure data produced by the OPA simulation of cascading transmission line overloads to:

1. Propose and analyze a statistical estimator of mean failure propagation λ that works in the presence of saturation.
2. Use estimates of initial failures and λ and the branching process model to predict the probability distribution of the number of transmission line failures and test these predicted distributions by comparing them with empirical distributions produced by the OPA simulation.
3. Evaluate the accuracy and reduced amount of data needed when using the estimators and branching process model to predict the probability distribution of the number of transmission line failures.

2. BRANCHING PROCESS WITH SATURATION

This section describes the branching process model used in this article.

2.1. Galton-Watson Branching Process

Suppose that there are N identical components and all components are initially unfailed. Component failures occur in stages, with Z_n the number of failures in stage n and Y_n the total number of failures up to and including stage n:

$$Y_n = Z_0 + Z_1 + Z_2 + \cdots + Z_n.$$

The process saturates when $S \leq N$ components fail. That is, if S components fail, the cascading process stops and there are no further failures.

There are Z_0 initial failures, where Z_0 has a Poisson distribution with parameter θ that is conditioned on a nonzero number of failures and has saturation at S failures:

$$P[Z_0 = r] = \begin{cases} \dfrac{e^{-\theta}}{1-e^{-\theta}} \dfrac{\theta^r}{r!}; & 1 \leq r < S \\ \dfrac{e^{-\theta}}{1-e^{-\theta}} \displaystyle\sum_{a=S}^{\infty} \dfrac{\theta^a}{a!}; & r = S. \end{cases} \quad (1)$$

Each of the Z_n failures in stage n independently produces a further number of failures in stage $n+1$ according to a Poisson distribution with mean λ, except that if the total number of failures exceeds S, then the total number of failures is limited to S. That is, the jth failure in stage n produces $Z_{n+1}^{[j]}$ failures in stage $n+1$ according to the Poisson distribution and the total number of failures in stage $n+1$ is:

$$Z_{n+1} = \min\left\{ Z_{n+1}^{[1]} + Z_{n+1}^{[2]} + \cdots + Z_{n+1}^{[Z_n]},\ S - Y_n \right\},$$

where $Z_{n+1}^{[1]}, Z_{n+1}^{[2]}, \ldots, Z_{n+1}^{[Z_n]}$ are independent. (A different form of saturation is described in References 11 and 12). The intent of the modeling with the branching process is not that each failure in each stage in some sense causes failures in the next stage; the branching process simply produces random numbers of failures in each stage that can statistically match the outcome of cascading processes.

We are interested in the total number of failures conditioned on a nonzero number of failures and this is distributed according to:

$$P[Y = r] = \begin{cases} \theta(r\lambda + \theta)^{r-1} \dfrac{e^{-r\lambda-\theta}}{r!(1-e^{-\theta})}; & 1 \leq r < S, \\ 1 - \displaystyle\sum_{s=1}^{S-1} \theta(s\lambda + \theta)^{s-1} \dfrac{e^{-s\lambda-\theta}}{s!(1-e^{-\theta})}; & r = S. \end{cases} \quad (2)$$

If there is no saturation ($S = \infty$) and $\lambda < 1$, then Equation (2) reduces to the generalized Poisson distribution[35,36] conditioned on nonzero failures:

$$P[Y = r] = \theta(r\lambda + \theta)^{r-1} \frac{e^{-r\lambda-\theta}}{r!(1-e^{-\theta})}; \quad 1 \leq r,$$

and the mean value of Y reduces to:

$$EY = \frac{\theta}{(1-\lambda)(1-e^{-\theta})}.$$

2.2. Saturation

There are several reasons for modeling saturation as described above so that there is a maximum of S components failed. Since there are always a finite number of components N, and $S \leq N$, saturation prevents more than N components failing. In the absence of saturation, there is a positive probability of an infinite number of components failing in the supercritical case of $\lambda > 1$. Thus, the saturation prevents nonphysical outcomes and allows the theory to apply to the supercritical case $\lambda > 1$.

Saturation with $S < N$ before all the components fail is a plausible effect in real blackouts that is not established or understood. Many observed cascading blackouts do not proceed to the entire interconnection blacking out. This may be due to the rarity of the largest blackouts or may be due to inhibition effects such as load shedding relieving system stress, or successful islanding in which the power system separates into disconnected portions. In any case, it is plausible that when cascading failure proceeds beyond a certain number of components, the cascading process will change its form due to the extreme degradation of the network and the modeling used for the initial part of the cascade will no longer be applicable. The methods of this article will apply if S is chosen to be this number of components. Defining the range of applicability of the cascading failure model in terms of the number of components failed using S is more plausible than defining it in terms of the number of stages of cascading.

Even if the saturation effects turn out in practice to be negligible in larger power networks (tens of thousands of nodes), much of the ongoing research on power system blackouts simulates much smaller power system networks with only hundreds of nodes, for pragmatic reasons. Some saturation effects have been observed in simulations of smaller power system networks[(12)] and saturation is one way to explain criticality phenomena observed in blackout simulations.[(7,20,21)]

In summary, at the present state of knowledge about cascading failure blackouts, there are several motivations and possible interpretations for modeling saturation. The methods of this article allow for saturation at S components failed and we avoid describing the cascading process after S components have failed and only estimate the propagation of failures before saturation is encountered.

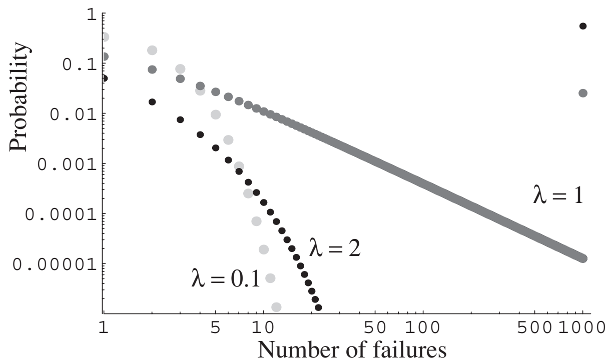

Fig. 1. Log–log plot of probability distribution of total number of failures Y in branching process model with saturation at $S = 1{,}000$ failures for three values of propagation λ. The distribution has an approximate power-law region at criticality when $\lambda = 1$. The probability of 1,000 failures is 0.025 for $\lambda = 1$, and 0.797 for the supercritical case $\lambda = 2$.

2.3. Behavior of Branching Model

If the parameters θ and λ can be estimated, then the saturating branching process predicts the probability distribution of total number of components failed according to Equation (2). This subsection briefly illustrates the qualitative behavior of the probability distribution of total number of components failed.

Assuming a mean initial failure of one ($\theta = 1$), Fig. 1 shows the probability distributions obtained from the saturating branching process for $S = N = 1{,}000$ and three values of λ. For subcritical $\lambda = 0.1$, well below 1, the probability of a large number near S of failures is exponentially small. The probability of exactly S failures is also very small. As λ increases in the subcritical range $\lambda < 1$, the mechanism by which there develops a significant probability of large number of failures near S is that the power-law region extends toward S failures.[(12)] For near critical $\lambda \approx 1$, there is a power-law region extending to S failures. For supercritical $\lambda = 2$, there is an exponential tail. This again implies that the probability of large number of failures $<S$ is exponentially small. However, there is a significant probability of exactly S failures that increases with λ.

3. ESTIMATING BRANCHING PROCESS PARAMETERS

3.1. Cascading Failure Simulation Data

Following the notation of Dobson *et al.*,[14] we suppose that the cascading failure simulation produces $Z_0 > 0$ initial failures in stage 0 and then iterates to produce further numbers of failures $Z_1,\ Z_2,\ Z_3, \ldots$ in stages $1, 2, 3, \ldots$, respectively. The assumption of $Z_0 > 0$ implies that all statistics are conditioned on the start of a cascade. The simulation is run K times to produce K independent realizations of the cascade. The failures in the kth run are written as $Z_0^{(k)},\ Z_1^{(k)},\ Z_2^{(k)},\ Z_3^{(k)}, \ldots$. The simulation results can be tabulated as follows:

	Stage 0	Stage 1	Stage 2	Stage 3	$\cdots$
Run 1	$Z_0^{(1)}$	$Z_1^{(1)}$	$Z_2^{(1)}$	$Z_3^{(1)}$	$\cdots$
Run 2	$Z_0^{(2)}$	$Z_1^{(2)}$	$Z_2^{(2)}$	$Z_3^{(2)}$	$\cdots$
Run 3	$Z_0^{(3)}$	$Z_1^{(3)}$	$Z_2^{(3)}$	$Z_3^{(3)}$	$\cdots$ (3)
.	.	.	.	.	
.	.	.	.	.	
Run K	$Z_0^{(K)}$	$Z_1^{(K)}$	$Z_2^{(K)}$	$Z_3^{(K)}$	$\cdots$

Define the cumulative number of failures in run k up to and including stage n as:

$$Y_n^{(k)} = Z_0^{(k)} + Z_1^{(k)} + Z_2^{(k)} + \cdots + Z_n^{(k)}.$$

3.2. Standard Propagation Estimator $\widehat{\lambda}_n$

The standard Harris estimator $\widehat{\lambda}_n$ of the offspring mean is:

$$\widehat{\lambda}_n = \frac{\sum_{k=1}^{K} \left(Y_n^{(k)} - Z_0^{(k)}\right)}{\sum_{k=1}^{K} Y_{n-1}^{(k)}}$$

$\widehat{\lambda}_n$ uses a fixed number of stages n for each run.

If there is no saturation, then $\widehat{\lambda}_n$ has some good statistical properties. In particular, $\widehat{\lambda}_n$ is a maximum likelihood estimator,[10,37,38] and a strongly consistent and asymptotically unbiased estimate of λ as $K \to \infty$.[39] Moreover, using the approach of Yanev,[39] $\widehat{\lambda}_n$ has an asymptotically normal distribution with variance:

$$\sigma^2(\lambda_n) = \frac{\lambda(1-\lambda)(1-e^{-\theta})}{K(1-\lambda^n)\theta}. \tag{4}$$

However, if there is saturation, then $\widehat{\lambda}_n$ becomes biased and underestimates λ as $K \to \infty$. This asymptotic bias is proved in the Appendix. The bias arises because stages of the branching process that encounter saturation usually have fewer failures. The bias of $\widehat{\lambda}_n$ is illustrated in Fig. 2 by generating data from the saturating branching process and evaluating the sample mean of $\widehat{\lambda}_n$ for a range of values of λ.

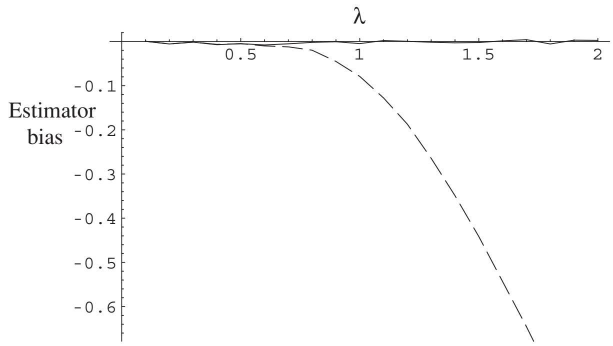

Fig. 2. Bias of estimator $\widehat{\lambda}_s$ (solid line close to horizontal axis) and bias of standard estimator $\widehat{\lambda}_n$ (dashed line) plotted against λ from saturating branching process with $\theta = 1$ and saturation $S = 20$. The bias is estimated from 1,000 samples of the estimator for $K = 50$ nonzero cascades.

The next subsection proposes a modified estimator with no asymptotic bias.

3.3. Propagation Estimator $\widehat{\lambda}_s$ that Accounts for Saturation

Each run of the simulation has a stage at which the number of failures is zero and remains zero for all subsequent stages, either because the cascade dies out, or the saturating number S of components have failed. Define:

$$s(k, S) = \max\left\{ n \,\middle|\, Y_n^{(k)} < S \text{ and } Z_{n-1}^{(k)} > 0 \right\}.$$

Then $s(k, S)$ is either the first stage at which there are zero failures or the last stage before a total of S failures.

We define the estimator of λ as:

$$\widehat{\lambda}_s = \frac{\sum_{k=1}^{K} \left(Z_1^{(k)} + Z_2^{(k)} + \cdots + Z_{s(k,S)}^{(k)}\right)}{\sum_{k=1}^{K} \left(Z_0^{(k)} + Z_1^{(k)} + \cdots + Z_{s(k,S-1)-1}^{(k)}\right)} = \frac{\sum_{k=1}^{K} \left(Y_{s(k,S)}^{(k)} - Z_0^{(k)}\right)}{\sum_{k=1}^{K} Y_{s(k,S-1)-1}^{(k)}}. \tag{5}$$

$\widehat{\lambda}_s$ only uses information from stages before saturation.

In the Appendix, we prove that $\widehat{\lambda}_s$ is strongly consistent, asymptotically unbiased, and normal as $K \to \infty$. The bias of $\widehat{\lambda}_s$ is illustrated in Fig. 2 for $K = 50$ nonzero cascades by generating data from the saturating branching process and evaluating the sample mean of $\widehat{\lambda}_s$. For small λ, $\widehat{\lambda}_s$ and $\widehat{\lambda}_n$ have almost the same bias because it is likely that cascades die out before reaching saturation. The Appendix also derives the variance of $\widehat{\lambda}_s$ in the subcritical case with no saturation and gives some numerical results for the bias and variance of $\widehat{\lambda}_s$.

3.4. Estimator $\widehat{\theta}$ of Mean Initial Failures

K samples of the initial failures are given by $Z_0^{(1)}, Z_0^{(2)}, \ldots, Z_0^{(K)}$. Let the sample mean of the initial failures be:

$$\overline{Z_0} = \frac{1}{K}\sum_{k=1}^{K} Z_0^{(k)}.$$

Then, neglecting saturation, both maximum likelihood and method of moments estimation of θ in Equation (1) yield an estimate $\widehat{\theta}$ satisfying:

$$\overline{Z_0} = g(\widehat{\theta}) = \frac{\widehat{\theta}}{1 - e^{-\widehat{\theta}}}. \tag{6}$$

(Accounting for saturation makes negligible difference to $\widehat{\theta}$ for $0 \le \theta \le 10$ and $S \ge 20$.) The variance of $\overline{Z_0}$ is, neglecting saturation:

$$\sigma^2(\overline{Z_0}) = \frac{\sigma^2(Z_0)}{K} = \frac{\theta(1 - e^{-\theta} - \theta e^{-\theta})}{K(1 - e^{-\theta})^2}$$

and hence, linearizing Equation (6), the variance of $\widehat{\theta}$ is:

$$\sigma^2(\widehat{\theta}) \approx \left(\frac{d(g^{-1})}{d\overline{Z_0}}\right)^2 \sigma^2(\overline{Z_0}) = \frac{\theta(1 - e^{-\theta})^2}{K(1 - e^{-\theta} - \theta e^{-\theta})}. \tag{7}$$

4. OPA SIMULATION OF CASCADING LINE FAILURES

The OPA simulation represents probabilistic cascading line failures in a power transmission network and is used to produce statistics such as the probability distribution of the number of line failures. This section summarizes the OPA simulation; for details see Reference 7 and for a larger context discussing cascading failure models in electric power systems see Reference 24.

At some network nodes, generators supply electric power that flows in the transmission lines according to circuit laws to substations at load nodes. The OPA model represents transmission lines, loads, and generators and computes the network power flows with the usual "DC load flow" approximation.[40] Each simulation run starts from a solved base case solution for the power flows and generation and loads that satisfy circuit laws and constraints. To obtain diversity in the runs, the system loads at the start of each run are varied randomly about their mean values by multiplying by a factor uniformly distributed in $[2 - \gamma, \gamma]$. γ determines the load variability. It is necessary and realistic to have some random variability in the model so that a range of possible cascades are simulated. γ is further discussed in References 7 and 32. Initial line failures are generated randomly by assuming that each line can fail independently with probability p_0. This crudely models initial line failures due to a variety of causes including lightning, wild fires, bad weather, and operational errors. Whenever a line fails, the generation and load is redispatched to satisfy the transmission line power flow constraints and generation constraints using standard linear programming methods (since there is more generation of power than the load requires, one must choose how to select and optimize the generation that is used to exactly balance the load in the network). The optimization cost function is weighted to ensure that load shedding is avoided where possible. If any lines were overloaded during the optimization, then these lines are lines that are likely to have experienced high stress, and each of these lines fails independently with probability p_1. The lines that have failed (if any), and other diagnostic data are recorded. The process of redispatch and testing for line failures is iterated until there are no more failures. The cascade of line failures continues in this manner until no further lines fail. Thus, the OPA simulation produces probabilistic cascading line failures in stages resulting from a random initial set of line failures.

The OPA model neglects many of the cascading processes in blackouts and the timing of failures. However, the OPA model does represent in a simplified way a dynamical process of cascading transmission line overloads and failures that is consistent with some basic network and operational constraints. In particular, the OPA simulation represents the simplified physics of power flow in the network of transmission lines as the lines successively reach their limits, probabilistically represents the tripping of the lines that reach their limits, and has a basic representation of generator redispatch and load shedding. We emphasize that OPA is much more complicated and entirely different than the branching process model that we are comparing it with. At the same time, we also emphasize that the OPA model is a highly simplified model of only one of the physical processes in power system cascading blackouts. Testing the branching process with OPA results does find out whether the branching process can reproduce one physical mechanism for cascading failure in an important application. But this testing does not establish that branching processes are generally applicable to other cascading failure processes in blackouts or in other applications.

5. RESULTS: PREDICTING THE DISTRIBUTION OF LINE FAILURES

We test the use of the estimators $\widehat{\lambda}_s$ and $\widehat{\theta}$ and the branching process model (Equation (2)) in predicting the distribution of line failures on cascading line outage data produced by the OPA simulation.

For each case considered, OPA was run so as to produce at least 5,000 cascading failures with a nonzero number of line failures. These cascades yield line failure data in the form of Equation (3). All the statistics are conditioned on a nonzero number of line failures. Then, $\widehat{\lambda}_s$ and $\widehat{\theta}$ are obtained using Equations (5) and (6). The number of cascades is large enough that the standard deviations of $\widehat{\lambda}_s$ and $\widehat{\theta}$ are negligible. (The influence on statistical accuracy of the small number of cascades desirable in practice is evaluated in the Appendix.)

The first three cases used the IEEE 118 node test system[41] at average load levels of 0.9, 1.0, and 1.3 times the base case loading. (The OPA parameters are $\gamma = 1.67$, $p_0 = 0.0001$, and $p_1 = 1$. These are typical values used in previous work and are more fully explained in Reference 7.) Since no saturation effects are observed in these results, we used a high-saturation value of $S = 100$ in Equation (5). The results are shown in Figs. 3–5 and Table I. The matches in Figs. 3–5 are very good. Fig. 5 shows a case with a large initial disturbance (the mean number of lines initially failed is estimated as $\widehat{\theta} \approx 12$).

The last four cases use the IEEE 300 node test system[41] at average load levels of 0.9, 1.0, 1.05, and 1.25 times the base case loading. (The OPA param-

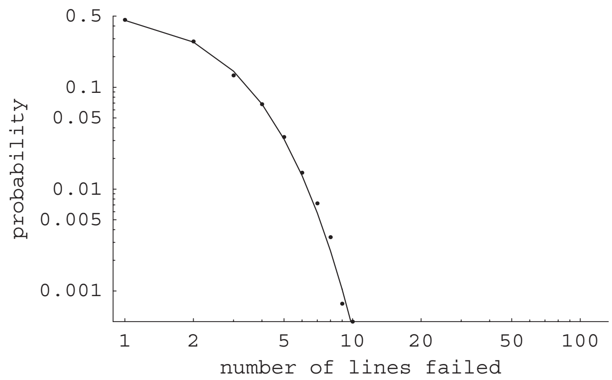

Fig. 3. IEEE 118 node test system with loading factor 0.9. Probability distribution of line outages estimated with branching process (solid line) compared with OPA empirical distribution (dots). Note the log–log scales. Reprinted with permission from Reference 29. © 2007 IEEE.

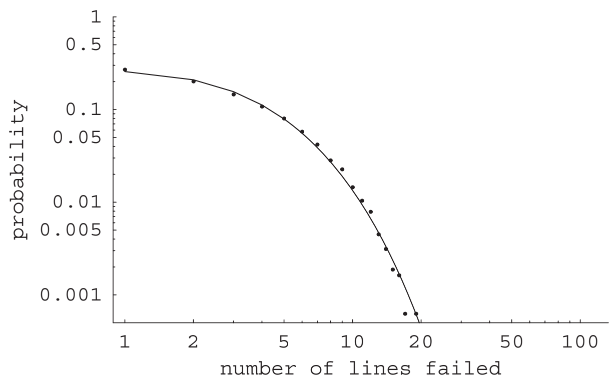

Fig. 4. IEEE 118 node test system with loading factor 1.0. Probability distribution of line outages estimated with branching process (solid line) compared with OPA empirical distribution (dots). Reprinted with permission from Reference 29. © 2007 IEEE.

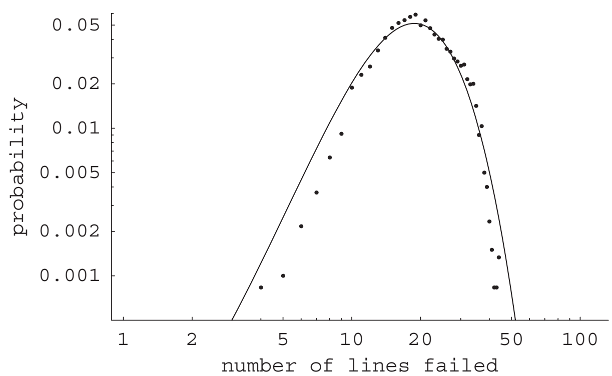

Fig. 5. IEEE 118 node test system with loading factor 1.3. Probability distribution of line outages estimated with branching process (solid line) compared with OPA empirical distribution (dots). Reprinted with permission from Reference 29. © 2007 IEEE.

Table I. Estimators $\widehat{\theta}$ and $\widehat{\lambda}_s$ for Simulation Cases

Power System	Loading Factor	$\widehat{\theta}$	$\widehat{\lambda}_s$
IEEE 118 node	0.9	1.10	0.19
IEEE 118 node	1.0	1.66	0.41
IEEE 118 node	1.3	12.2	0.44
IEEE 300 node	0.9	0.52	0.15
IEEE 300 node	1.0	0.81	0.26
IEEE 300 node	1.05	1.30	0.47
IEEE 300 node	1.25	4.56	0.65

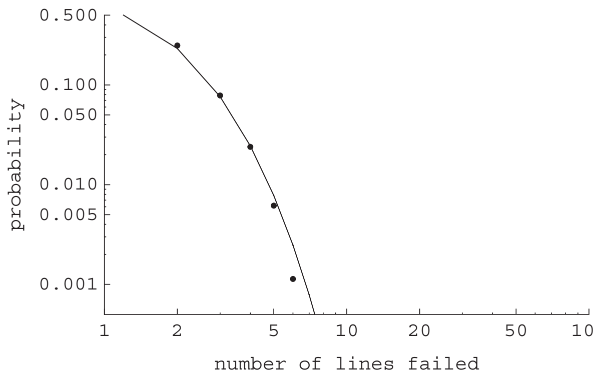

Fig. 6. IEEE 300 node test system with loading factor 0.9. Probability distribution of line outages estimated with branching process (solid line) compared with OPA empirical distribution (dots).

eters[3] are $\gamma = 1.67$, $p_0 = 0.001$, and $p_1 = 0.15$, and Equation (5) is evaluated with saturation $S = 100$.) The results are shown in Figs. 6–9 and Table I. The results in Figs. 6–8 show a good match. The highly stressed case of 1.25 times the base case loading in Fig. 9 shows a much poorer match, and we suspect that this is caused by additional lines being forced to trip in the first few stages.

The results show good predictions of the probability distributions of the number of line failures for all cases except for a highly stressed case of the IEEE 300 node test network. For further results on an artificial 190 node test network, see Reference 14.

Computing the empirical distributions is time consuming: it takes several days to compute the 5,000

[3] The ratios of each line flow limit to its base case line flow (not specified in the IEEE data) were determined by running OPA with self-organization that selectively upgrades lines in response to their participation in blackouts,(32) starting from an initial guess of the line limits. This procedure results in a coordinated set of line limits.

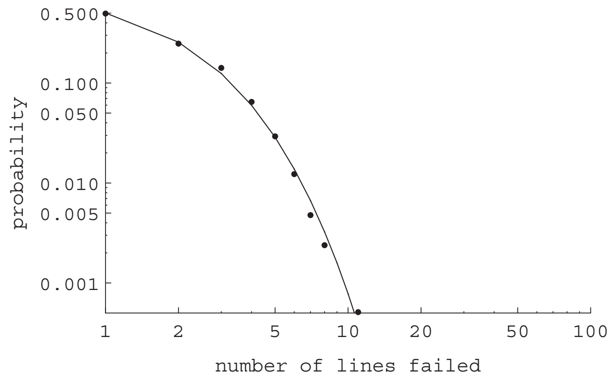

Fig. 7. IEEE 300 node test system with loading factor 1.0. Probability distribution of line outages estimated with branching process (solid line) compared with OPA empirical distribution (dots).

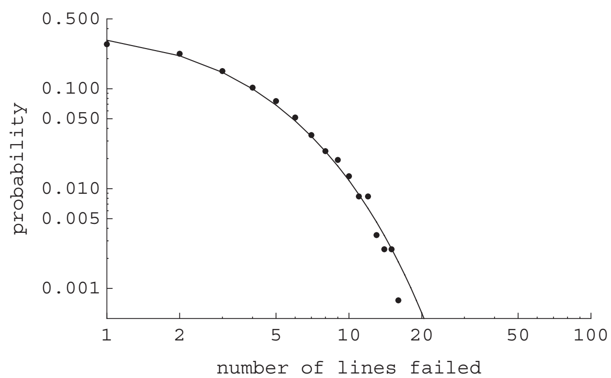

Fig. 8. IEEE 300 node test system with loading factor 1.05. Probability distribution of line outages estimated with branching process (solid line) compared with OPA empirical distribution (dots).

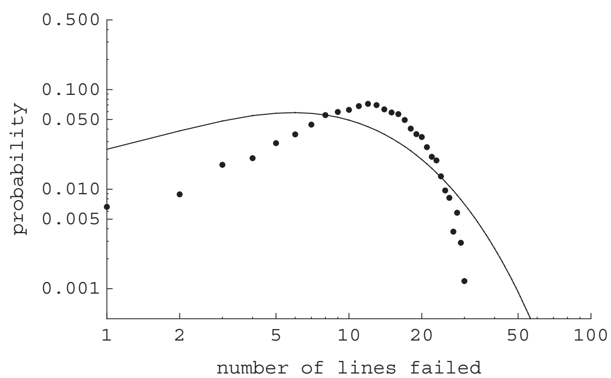

Fig. 9. IEEE 300 node test system with loading factor 1.25. Probability distribution of line outages estimated with branching process (solid line) compared with OPA empirical distribution (dots).

nonzero cascades for each of the 300 bus system cases. This reinforces the need for faster methods such as the estimation via a branching process model developed in this article.

6. NUMBER OF CASCADES VERSUS ACCURACY

This section estimates how many more cascades are needed to estimate the probability distribution of the total number of failures empirically than via the branching process model, assuming that the variance of the estimates is the same. The case considered is the total number of failures less than saturation ($1 \leq r < S$), large S, and $\lambda < 1$.

For $1 \leq r < S$, rewrite the probability distribution of the total number of failures from Equation (2) as:

$$p(r,\theta,\lambda) = P[Y = r] = \theta(r\lambda + \theta)^{r-1}\frac{e^{-r\lambda-\theta}}{r!(1-e^{-\theta})}. \tag{8}$$

The empirical estimate $\widehat{p}(r,\theta,\lambda)$ from M cascades is the number of cascades with r total failures divided by M. The empirical estimate $\widehat{p}(r,\theta,\lambda)$ has variance:

$$\sigma^2(\widehat{p}) = \frac{p(r,\theta,\lambda)[1-p(r,\theta,\lambda)]}{M}. \tag{9}$$

The estimate of the distribution $p(r,\theta,\lambda)$ from K cascades via the branching process is $p(r,\widehat{\theta},\widehat{\lambda}_s)$. The asymptotic variance of $p(r,\widehat{\theta},\widehat{\lambda}_s)$ can be obtained by linearizing Equation (8), approximating $\sigma^2(\widehat{\lambda}_s)$ by $\sigma^2(\widehat{\lambda}_\infty)$ for large saturation S, and substituting from Equation (A.2) and (7) to obtain:

$$\begin{aligned}\sigma^2[p(r,\widehat{\theta},\widehat{\lambda}_s)] &\approx \left(\frac{\partial p}{\partial \lambda}\right)^2\sigma^2(\widehat{\lambda}_s) + \left(\frac{\partial p}{\partial \theta}\right)^2\sigma^2(\widehat{\theta})\\ &\approx \left(\frac{\partial p}{\partial \lambda}\right)^2\frac{\lambda(1-\lambda)(1-e^{-\theta})}{K\theta}\\ &\quad + \left(\frac{\partial p}{\partial \theta}\right)^2\frac{\theta(1-e^{-\theta})^2}{K(1-e^{-\theta}-\theta e^{-\theta})}.\end{aligned} \tag{10}$$

The use of the asymptotic variance (Equation (A.2)) requires large S and $\lambda < 1$.

Equating the variances of Equations (9) and (10) so that $\sigma^2(\widehat{p}) = \sigma^2[p(r,\widehat{\theta},\widehat{\lambda}_s)]$ bounds the ratio of the number of cascades M needed to estimate $\widehat{p}$ and the number of cascades K needed to estimate $p(r,\widehat{\theta},\widehat{\lambda}_s)$.

$$\frac{M}{K} \gtrsim B\,,$$

where

$$B = \frac{p(r,\theta,\lambda)[1-p(r,\theta,\lambda)]}{\left(\frac{\partial p}{\partial \lambda}\right)^2\frac{\lambda(1-\lambda)(1-e^{-\theta})}{\theta} + \left(\frac{\partial p}{\partial \theta}\right)^2\frac{\theta(1-e^{-\theta})^2}{(1-e^{-\theta}-\theta e^{-\theta})}}. \tag{11}$$

Evaluating Equation (11) numerically gives the minimum M/K as a function of r, θ, and λ. For example, evaluating Equation (11) for $\theta = 1$ and $0 < \lambda < 1$ shows that $B > 10$ for $r \geq 8$ and $B > 100$ for $r \geq 55$. Evaluating Equation (11) for $\theta = 5$ and $0 < \lambda < 1$ shows that $B > 10$ for $r \geq 11$ and $B > 100$ for $r \geq 70$. Evaluating Equation (11) for $\theta = 10$ and $0 < \lambda < 1$ shows that $B > 10$ for $r \neq 6$ and $B > 100$ for $r \geq 82$. These results indicate that the estimation of $p(r,\theta,\lambda)$ via the branching process compared to the empirical estimation of $p(r,\theta,\lambda)$ requires one or two orders of magnitude fewer cascades for moderate or large r to achieve the same variance.

7. CONCLUSION

In this article, we approximate cascading failure by a Galton-Watson branching process with saturation in order to propose a method of quantifying the mean propagation of failures λ. The proposed estimator $\widehat{\lambda}_s$ for λ requires multiple observations of cascades with the initial and final failures grouped in stages. Unlike the standard Harris estimator of λ, the estimator $\widehat{\lambda}_s$ has zero asymptotic bias in the presence of saturation modeled by a maximum number of components failed.

The branching process model gives an analytic formula to predict the distribution of the total number of failures from an estimate $\widehat{\theta}$ of the mean initial failures and the estimate $\widehat{\lambda}_s$ of the mean propagation. We test this prediction on cascading failure data from a simulation of cascading transmission line outages in standard IEEE electric power system test networks of 118 and 300 nodes. The predicted distribution is close to the empirical distribution of total number of line outages for all cases except for a highly stressed case on the 300 node test system. That is, except for this case, the joint use of the estimators $\widehat{\theta}$, $\widehat{\lambda}_s$, and the branching process model is effective in predicting the distribution of the total number of line outages.

Since the simulation used for testing the branching process approximately models only one of the physical mechanisms involved in cascading blackouts, we are not able at present to draw general conclusions about the extent to which branching processes capture other mechanisms of cascading failure in blackouts. However, the results of this article are sufficiently promising to indicate that future work testing the branching process model on other cascading failure simulations or observed blackout data would be worthwhile. For example, the branching process model could be tested on data generated by more detailed cascading failure simulations such as the Manchester model[42] or TRELSS.[43] The approach could also be tested on cascading failure data for large networked infrastructures or in interacting infrastructures. If future testing on simulated and real data succeeds in establishing branching process models for cascading failure, we note that the ability to estimate the propagation of failures and the distribution of failures with a modest number of observations would expand the opportunities for using cascading failure simulations to study the effect of transmission system upgrades on cascading failure and would be crucial for the practicality of monitoring failures in the power grid to assess the overall risk of cascading failure.

We estimate the minimum number of cascades to be simulated in order to get sufficiently accurate estimates of λ and the probability distribution of the total number of failures. For example, with saturation at a large number of components, 50 cascades yield $\widehat{\lambda}_s$ with negligible bias and worst-case standard deviation less than 0.06. Direct empirical estimation of the distribution of the total number of failures in the region of a moderate or large number of components failed requires one to two orders of magnitude more cascades than first estimating θ and λ and then using the branching process model to predict the distribution of the total number of failures.

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APPENDIX: BIAS AND VARIANCE OF $\widehat{\lambda}_s$

We continue to assume that the branching process has Poisson initial failures with mean θ and a Poisson offspring distribution with mean λ.

To show that $\widehat{\lambda}_s$ is asymptotically unbiased, rewrite Equation (5) as:

$$\widehat{\lambda}_s = \frac{\dfrac{1}{K}\displaystyle\sum_{k=1}^{K}\sum_{i=0}^{S-3} Z_{i+1}^{(k)} I\left[Y_{i+1}^{(k)} < S\right]}{\dfrac{1}{K}\displaystyle\sum_{k=1}^{K}\sum_{i=0}^{S-3} Z_{i}^{(k)} I\left[Y_{i+1}^{(k)} < S-1\right]}.$$

Let

$$w_i^{(k)} = Z_{i+1}^{(k)} I\left[Y_{i+1}^{(k)} < S\right] - \lambda Z_i^{(k)} I\left[Y_{i+1}^{(k)} < S-1\right].$$

Then

$$\widehat{\lambda}_s - \lambda = \frac{\dfrac{1}{K}\displaystyle\sum_{k=1}^{K}\sum_{i=0}^{S-3} w_i^{(k)}}{\dfrac{1}{K}\displaystyle\sum_{k=1}^{K} Y_{s(k,S-1)-1}^{(k)}}. \tag{A.1}$$

For each k, $Y_{s(k,S-1)-1}^{(k)}$ is bounded by S and has finite mean and variance. Moreover, $Y_{s(k,S-1)-1}^{(k)}$, $k = 1, 2, \ldots, K$ are independent and the strong law of large numbers implies that the denominator of Equation (A.1) tends almost surely to a constant.

Therefore, to prove that $E(\widehat{\lambda}_s - \lambda) \to 0$ almost surely and $\widehat{\lambda}_s$ is asymptotically unbiased, it is sufficient to show that $Ew_i^{(k)} = 0$ for $i = 0, 1, 2, \ldots, S-3$. But $Ew_i^{(k)} = 0$ follows from:

$$\begin{aligned}
&E[Z_{i+1} I(Y_{i+1} < S)]\\
&\quad = E[E[Z_{i+1} I[Z_{i+1} < S - Y_i] | Y_i, Z_i]]\\
&\quad = E\left[\sum_{m=1}^{S-Y_i-1} m \frac{(Z_i\lambda)^m}{m!} e^{-Z_i\lambda}\right]\\
&\quad = \lambda E\left[\sum_{m=0}^{S-Y_i-2} Z_i \frac{(Z_i\lambda)^m}{m!} e^{-Z_i\lambda}\right]\\
&\quad = \lambda E\big[E[Z_i I[Y_i + Z_{i+1} < S-1] | Y_i, Z_i]\big]\\
&\quad = \lambda E[Z_i I[Y_{i+1} < S-1]].
\end{aligned}$$

We derive the asymptotic variance of $\widehat{\lambda}_s$ in the subcritical case of $\lambda < 1$ and when saturation is neglected by letting $S \to \infty$. When $\lambda < 1$, the branching process dies out with $Z_i^{(k)} \to 0$ as $i \to \infty$ almost surely and $Y_n^{(k)} \to Y_\infty^{(k)}$ as $n \to \infty$ almost surely.

Table A1. Bias and Standard Deviation of $\widehat{\lambda}_s$ on Saturating Branching Process with $\theta = 1$

Number of Runs K	Saturation S	Bias $\max_{0<\lambda<2} \lvert\mu(\widehat{\lambda}_s) - \lambda\rvert$	Standard Deviation $\max_{0<\lambda<2} \sigma(\widehat{\lambda}_s)$
10	20	0.035	$0.28 = 0.87/\sqrt{K}$
20	20	0.018	$0.18 = 0.80/\sqrt{K}$
50	20	0.008	$0.11 = 0.78/\sqrt{K}$
200	20	0.004	$0.055 = 0.77/\sqrt{K}$
10	100	0.050	$0.16 = 0.57/\sqrt{K}$
20	100	0.027	$0.092 = 0.41/\sqrt{K}$
50	100	0.010	$0.057 = 0.40/\sqrt{K}$
200	100	0.003	$0.029 = 0.41/\sqrt{K}$

Hence, the Harris estimator $\widehat{\lambda}_n \to \widehat{\lambda}_\infty$ as $n \to \infty$, where:

$$\widehat{\lambda}_\infty = \frac{\sum_{k=1}^{K} \left(Y_\infty^{(k)} - Z_0^{(k)} \right)}{\sum_{k=1}^{K} Y_\infty^{(k)}}.$$

Moreover, for $\lambda < 1$, our estimator $\widehat{\lambda}_s \to \widehat{\lambda}_\infty$ as $S \to \infty$. From Equation (4), the variance of $\widehat{\lambda}_\infty$ as $K \to \infty$ is:

$$\sigma^2(\widehat{\lambda}_\infty) = \frac{\lambda(1-\lambda)(1-e^{-\theta})}{K\theta}. \tag{A.2}$$

Thus, Equation (A.2) gives the asymptotic variance of $\widehat{\lambda}_s$ as $K \to \infty$ and $S \to \infty$ for $\lambda < 1$. For example, for $\theta = 1$, the maximum asymptotic variance occurs for $\lambda = 0.5$ and the asymptotic standard deviation from Equation (A.2) becomes $\sigma(\widehat{\lambda}_\infty) = 0.40/\sqrt{K}$.

To augment these asymptotic results, the estimator $\widehat{\lambda}_s$ is tested on the saturating branching process with $\theta = 1$ and $0 < \lambda < 2$. The worst-case bias and standard deviation of $\widehat{\lambda}_s$ are determined numerically from 1,000 cascades with nonzero failures and the results are shown in Table A1. The asymptotic variance (Equation (A.2)) and Table A1 can be used to estimate the number of cascades K needed to obtain a given standard deviation in $\widehat{\lambda}_s$.

The proof that the standard estimator $\widehat{\lambda}_n$ is asymptotically unbiased when there is no saturation relies on the fact that $E[Z_{i+1}] = \lambda E[Z_i]$.[39] When there is saturation, $\widehat{\lambda}_n$ asymptotically underestimates λ because the following shows that $E[Z_{i+1}] < \lambda E[Z_i]$.

$$\begin{aligned}
E[Z_{i+1}] &= E[E[Z_{i+1}|Y_i, Z_i]] \\
&= E\left[\sum_{r=1}^{S-Y_i-1} r \frac{(Z_i\lambda)^r}{r!} e^{-Z_i\lambda} \right. \\
&\qquad \left. + (S - Y_i) \sum_{r=S-Y_i}^{\infty} \frac{(Z_i\lambda)^r}{r!} e^{-Z_i\lambda} \right] \\
&= E\left[\sum_{r=1}^{\infty} r \frac{(Z_i\lambda)^r}{r!} e^{-Z_i\lambda} \right. \\
&\qquad \left. - \sum_{r=S-Y_i}^{\infty} (r - S + Y_i) \frac{(Z_i\lambda)^r}{r!} e^{-Z_i\lambda} \right] \\
&< \lambda E\left[Z_i \sum_{r=0}^{\infty} \frac{(Z_i\lambda)^r}{r!} e^{-Z_i\lambda} \right] \\
&= \lambda E[Z_i].
\end{aligned}$$

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